A quantitative evaluation of veto power

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Abstract In this paper we introduce the decisiveness index and the loose protectionism index for a single player, starting form the decisiveness and the loose protectionism indices for a collective decision-making mechanism defined by Carreras. The attention is mainly focused on the second one, which is proposed as a quantitative measure of the power of veto of each agent. According to this index, a veto player has veto power equal to one, while each other player has a fraction according to her/his possibility to block a given proposal. Such an index coincides with the expected payoff at the Bayesian equilibrium of a suitable Bayesian game, which catch the non-cooperative point of view of a decision-making mechanism.

Keywords Veto power, Indices, Quantitative measure, Bayesian game

1 Introduction

The United Nations Security Council (UNSC) represents the most typical example of assembly where some actors are endowed with veto power. It is composed of 5 permanent members (a protecting philosophy brought to the designation of the five winner countries of the World War II, China, France, Russian Federation, the United Kingdom and the United States, during the postwar period) and 10 non-permanent members for two-year terms starting on 1 January, with five replaced each year. Each Council member has one vote and decisions on procedural matters are made by an affirmative vote of at least 9 of the 15 members. Also decisions on substantive matters require 9 votes, but the concurring votes of all 5 permanent members are needed. This rule, called *great Power unanimity* is often simply referred to as the

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veto power and the 5 permanent members are called *veto players* [2].

Among others, functions and powers of the UNSC under the Charter are to maintain international peace and security in accordance with the principles and purposes of the United Nations, which is the primary responsibility, but also to investigate any dispute or situation which might lead to international friction, to take military action against an aggressor, etc. The UNSC alone has the power to take decisions which Member States are compelled to carry out under the Charter.

As it is observed by Mercik [14], quite intuitively the right of veto will increase the power of a player in most cases. This is exactly the case of the UNSC, where the five permanent members have a large power in the decision process of substantive matters, not only because of the absence of a term-period, but mainly because of their right to veto.

According to the definition of Tsebelis [21] in his *veto players' theory*, a veto player is an individual or collective actor whose agreement is necessary for policy changes. The *policy stability*, i.e. the impossibility of significant changes of the *status quo*, is strictly related to the role of veto players, as a significant policy change has to be approved by all of them. In his large literature on the topic, Tsebelis analyzes the connections between veto players and other important features: the agenda control or the production of significant laws, for example, are strictly related to this topic and veto players' theory represents for Tsebelis a way to unify the understanding of politics.

Stated that the power of veto represents a central topic in politics, it is natural to ask: how to evaluate it? This question brought, in the last years, to an increasing in the number of papers and surveys on the topic, but the attention to veto power indices in the literature is still less than that devoted to power indices. Two questions arise at first: are veto power and power analogous concepts? May we evaluate them with the same instruments? The wide range of existing power indices takes into account different features, like the ordering of the players in a coalition (Shapley-Shubik index [20]), the different majorities (Banzhaf-Coleman index [1] and [5]), the importance of the minimal winning coalitions (Public Good Index [9] and Deegan-

[2] For further details on the UNSC see http://www.un.org/Docs/sc/index.html.

Packel index [6]), and of the quasi-minimal winning coalitions (Johnston index [11]), the weights of the players (Weighted Shapley value [12]). In order to better study political situations, the possible ideological affinities have been analyzed, accounting possible existing relations among the players (Owen value [18]) or drawing graphs which represent the possible connections between the players (Myerson index [16] and FP indices [7] and [4]).

Our idea is that some different features have to be considered in order to define an index suitable for analyzing the power of veto. A party, for example, can be able alone to block a proposal voting against it, but it may not have the possibility to make an opposite law being approved without the support of other parties; this happens, for example, to a permanent member of the UNSC, which has full veto power, but not full power according to the classical indices. Moreover, the concepts of *a priori unions* ([18], [19]) and/or of *connected coalitions* ([16], [7], [4]), which have been introduced to better represent the relations between parties, are no longer relevant while speaking about the power of a party which is against the approval of a proposal. In order to block a proposal it is not necessary anymore to have a common ideological position: two parties very far from each other can be both, because of opposite reasons, decide to vote against a law, even if this does not mean that they would agree in making a common different proposal being approved.

In the work of Carreras ([2] and [3]) there is a large analysis of the blocking power in a simple game, both from a collective and from an individual point of view. The *strict protectionism index* and the *Banzhaf strict blocking power index* are proposed as suitable tools to deal with the problem. The *blockability relation* have been formalized by Ishikawa and Inohara [10] and just a couple of years after, Kitamura and Inohara [13] propose the *blockability index* which is a power index for coalitions. Another important contribution to the problem is given by Mercik [15], who studies the problem of evaluating the power of veto starting from Johnston power index, suggested after defining some suitable axioms that an index for veto should satisfy.

In this paper, we present a quantitative approach to the problem of evaluating the power of veto. We start from the observation that it is not necessary anymore that the power of the agents sum up to a given fixed number, which is normally assumed to be equal to 1, as one, two, or all the agents of a voting procedure may have full power to block a proposal. We will define an index which assigns full veto power (for simplicity equal to 1) to all those agents who are able alone to veto a proposal; all the other players will be given a non-negative veto power smaller than 1 according to their possibility to stop an approval joining other players. After some preliminaries in Section 2, we recall the work of Carreras in Section 3, from which the idea of this paper is taken, and we present the proposed index to evaluate veto power in Section 4. In Section 5 we give a Bayesian formulation of the model, which is a good way to deal with the problem by a non-cooperative point of view. Section 6 concludes.

2 Preliminaries

A *cooperative TU game* is a pair (N, v) where $N = \{1, 2, \ldots, n\}$ is a nonempty finite set of players and $v : 2^N \to \mathbb{R}$ is the *characteristic function* which associates with every nonempty subset S of N, called *coalition*, a real number $v(S)$, which represents the *worth* of S. (N, v) is *simple* if $v(S) \in \{0, 1\}$ for each $S \subseteq N$, $v(N) = 1$ and given $S, T \subseteq N$, $S \subseteq T \Rightarrow v(S) \leq v(T)$. When $v(S) = 1$ we say that the coalition is *winning*, *losing* otherwise. For further details we address the reader to the book by Osborne and Rubinstein [17].

We call W the set of all the winning coalitions. A *veto player* is a player i that belongs to all winning coalitions, i.e. for each $S \in W$, then $i \in S$. A *dictator* is a player that is winning without any support, i.e. $\{i\} \in W$. Given a coalition $S \in W$, a player $j \in S$ is *critical* for S if $S \setminus \{j\} \notin W$. The quantity $v(S) - v(S \setminus \{j\})$ is called the *marginal contribution of player j w.r.t. S*. We say a winning coalition is *minimal* if each proper subcoalition is losing and we call W^m the set of all the minimal winning coalitions.

Political situations are frequently described by simple games, which are often defined through weighted majority situations. Let N be the set of parties of a Parliament, in our model the players of the game. A *vector of weights* $w = (w_1, w_2, \dots, w_n)$ is associated to N, where w_i, $i \in N$ is a non negative weight given to each party that may represent, for example, the number of seats it has. Fixing a *majority quota* q, i.e. the number of votes needed in order to pass a proposal, we obtain a *weighted majority situation* denoted by $[q; w_1, w_2, \dots, w_n]$. Given a weighted majority situation it is possible to define the corresponding *weighted majority game* (N, v), where the characteristic function $v : 2^N \to \{0, 1\}$ is

$$v(S) = \begin{cases} 1 & \text{if } \sum_{j \in S} w_j \geq q \\ 0 & \text{otherwise} \end{cases} \qquad \forall S \subseteq N$$

A simple game can always be described by the set W (or simply the set W^m), and in this case it is denoted as (N, W), while it is not always possible to represent it as a weighted majority situation. Defining the game directly by giving the set of its winning coalitions allows painting a greater range of scenarios in which, for example, a player is endowed with veto power independently from the number of seats, as it happens for the UNSC.

3 Decisiveness and loose protectionism indices

In this section we recall the work of Carreras [2], that is mainly based on the idea of providing a numerical measure of the agility of the collective decision-making mechanism. Given a game (N, W), the set of coalitions 2^N splits into four classes, namely:

- D (*decisive winning*): $S \in W$ such that $N \setminus S \notin W$;
- C (*conflictive winning*): $S \in W$ such that $N \setminus S \in W$;
- Q (*blocking*): $S \notin W$ such that $N \setminus S \notin W$;
- P (*strictly losing*): $S \notin W$ such that $N \setminus S \in W$.

Thus, $W = D \cup C$. The family Q is called the *blocking family*; the game is *strong* if $Q = \emptyset$ and *weak* otherwise. The game is *proper* if $C = \emptyset$ and *improper* otherwise. When a game is proper and strong, it is called *decisive*.

Carreras defines the *decisiveness index* of the game (N, W) as

$$\delta(N, W) = \frac{|W|}{2^n}$$

where $n = |N|$. It gives the probability that an abstract proposal will pass in (N, W), where each agent $i \in N$ has only two options [3]: voting for the proposal (Y) or voting against it (N), with probability 1/2. We remark that the probabilities of the agents are assumed to be independent. The motion will pass if and only if the set of agents that vote for Y is a winning coalition $S \in W$. Obviously $0 < \delta(N, W) < 1$ as $\emptyset \notin W$ and $N \in W$. Given two simple games with the same player set, $|W| < |W'|$ implies $\delta(N, W) < \delta(N, W')$. If a game is decisive, then $\delta(N, W) = 1/2$ independently of the number of involved players. Since no improper and weak weighted majority game exists, in this subclass the index 1/2 characterizes the decisive games. Carreras also observes that when a game is weak and proper, the decisiveness index is smaller than 1/2 and when it is improper and strong the index is greater than 1/2.

Let (N, W) be a simple game and (N, W^*) be the dual game where $W^* = \{S \subseteq N : N \setminus S \notin W\}$. Thus, $W^* = D \cup Q$ and

1. $\delta(N, W^*) + \delta(N, W) = 1$
2. $\delta(N, W^*) - \delta(N, W) = \frac{|Q|-|C|}{2^n}$

The dual game allows defining an obvious protectionism index, which Carreras calls the *loose protectionism index*, based on the idea of providing a numerical measure of the inertia of any decision-making mechanism; in formula

$$\delta^*(N, W) = \delta(N, W^*) = 1 - \delta(N, W)$$

It gives the probability that a proposal will not pass in (N, W), where again each agent has the options to vote for or against the proposal, with probability 1/2. This index is suggested as a possible choice to define a collective blocking index for simple games to measure the blocking capability at collective level, analyzing the game by a protectionism viewpoint. As $\delta^*(N, W) = 1 - \delta(N, W)$, in [3] Carreras observes that it does not provide any new information, preferring to adopt an index based

[3] the abstention is allowed, but it counts for "against".

on the strict notion of blocking. The *strict protectionism index* is then defined as $\pi(N, W) = |Q|/2^n$.

Then he defines a *blocking swing* for player $i \in N$ as a pair $(S, S \setminus \{i\})$ s.t. $S \in Q$ and $S \setminus \{i\} \notin Q$ and he calls the *Banzhaf strict protectionism index* an individual blocking power index for the players involved defined as

$$\rho_i(N, W) = \xi_i(N, W)/2^{n-1}$$

where $\xi_i(N, W)$ is the number of blocking swings for player i.

4 Decisiveness and loose protectionism indices of a player

Now, we introduce a new index to measure the decisiveness of a game from the perspective of each player, which gives the probability that a proposal will pass in (N, W) when we already know that player i votes in favor of the proposal (Y) and the other players vote in favor or against with probability 1/2. Let W_i be the set of winning coalitions including player i, i.e. $W_i = \{S \in W : i \in S\}$.

Definition 1. *The* decisiveness index of player i *is defined as*

$$\delta_i(N, W) = \frac{|W_i|}{2^{n-1}} \tag{1}$$

Obviously, $0 < \delta_i(N, W) \leq 1$ as $N \in W_i$. If the outcome A stands for "the proposal will pass" and B for "player i votes in favor", the index corresponds to the conditional probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Proposition 1. *When i is a veto player,* $\delta_i(N, W) = 2\delta(N, W)$

Proof Writing the conditional probability as $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$ and considering that $P(B) = 1/2$, $P(A|B) = \delta_i(N, W)$ and $P(A) = \delta(N, W)$, we obtain $\delta_i(N, W) = 2P(B|A)\delta(N, W)$. When i is a veto player $P(B|A) = 1$ then $\delta_i(N, W) = 2\delta(N, W)$.

■

Similarly, we define the probability that a proposal will not pass in (N, W) when we know that player i votes against the proposal (N) and the others vote in favor or against with probability 1/2.

Definition 2. *The* loose protectionism index of player i *is defined as*

$$\delta_i^*(N, W) = \frac{2^{n-1} - |W| + |W_i|}{2^{n-1}} = 1 - 2\delta(N, W) + \delta_i(N, W) \tag{2}$$

Obviously, $0 < \delta_i^*(N, W) \leq 1$ as $\emptyset \notin W$. The numerator counts the number of losing coalitions which do not include player i, i.e. the number of coalitions without player i voting in favor of the proposal but not being able to make it approved.

Remark 1. *We may observe that player i is a dictator iff $\delta_i(N, W) = 1$ and is of veto iff $\delta_i^*(N, W) = 1$. The indices $\delta_i(N, W)$ and $\delta_i^*(N, W)$ are strictly related and this relation depends on the decisiveness index of the game. In particular*

- *if the game is weak and proper, as $\delta(N, W) < 1/2$, we get that $\delta_i^*(N, W) > \delta_i(N, W)$ for each $i \in N$;*
- *if the game is strong and improper, by duality $\delta(N, W) > 1/2$ and $\delta_i^*(N, W) < \delta_i(N, W)$ for each $i \in N$;*
- *if the game is decisive, then a player is of veto ($\delta_i^*(N, W) = 1$) iff it is a dictator ($\delta_i(N, W) = 1$), in general when the game is decisive, $\delta_i(N, W) = \delta_i^*(N, W)$ for each $i \in N$. In a weighted majority game, as there are no weak and improper games, we are always able to say if the players will have a higher power or a higher power of veto, or if they are equivalent.*

The two indices are also directly related as stated in the following proposition.

Proposition 2. *$\delta_i^*(N, W) = \delta_i(N, W^*)$, for every $i \in N$.*

Proof We have that $\delta_i(N, W^*) = \frac{|W_i^*|}{2^{n-1}}$ and $\delta_i^*(N, W) = \frac{2^{n-1} - |W| + |W_i|}{2^{n-1}}$.

Let $D_i = \{S \in D : i \in S\}$, $C_i = \{S \in C : i \in S\}$, $Q_i = \{S \in Q : i \in S\}$ and $P_i = \{S \in P : i \in S\}$. We want to show that

$$2^{n-1} - |W| + |W_i| = |W_i^*|$$

i.e.

$$|D_i| + |C_i| + |Q_i| + |P_i| - |D| - |C| + |D_i| + |C_i| = |D_i| + |Q_i|$$

$$|D_i| + 2|C_i| + |P_i| = |D| + |C|$$

and this is true as $|D_i| + |P_i| = |D|$ and $2|C_i| = |C|$. In fact

$$\begin{aligned} |D_i| + |P_i| &= |\{S \in W,\ i \in S:\ N \setminus S \notin W\}| + |\{S \notin W,\ i \in S:\ N \setminus S \in W\}| \\ &= |\{S \in W:\ N \setminus S \notin W\}| = |D| \end{aligned}$$

and

$$|C_i| = |\{S \in W,\ i \in S:\ N \backslash S \in W\}| = |\{S \in W,\ i \notin S:\ N \backslash S \in W\}| = |C \backslash C_i| = |C| - |C_i|$$

■

As observed in [2], the Banzhaf index [1] also measures the decisiveness of a game from the perspective of each player. In particular, let (N, W) be a game and $i \in N$, then

$$\beta_i(N, W) = 2\delta(N, W) - 2\delta(N_{-\{i\}}, W_{-\{i\}}).$$

where $(N_{-\{i\}}, W_{-\{i\}})$ denotes the residual game that arises when player i leaves, i.e. the subgame with players set $N \setminus \{i\}$. This basic relationship between the decisiveness index and the Banzhaf index suggests us to look for a possible relation between the Banzhaf index and the indices defined in (1) and (2). When $i \in N$ is a veto player, as Carreras noticed $\beta_i(N, W) = 2\delta(N, W)$, then by Proposition 1 we simply get that $\beta_i(N, W) = \delta_i(N, W)$.

The indices in (1) and (2) are quantitative indices and they evaluate the power of a player in making a proposal been accepted (δ_i) or rejected (δ_i^*).

In Example 1 we compute the previous indices for a simple theoretical situation. For sake of completeness, we add the Banzhaf strict protectionism index, ρ, proposed by Carreras [3], and Johnston index [11], J, as suggested by Mercik [15]. The comparison is carried out using the ratios of the indices among the players, as not all the indices sum up to one.

Example 1. *Consider the simple weighted majority situation* $[6; 2, 3, 5]$ *representing a Parliament with only three parties, then* $N = \{1, 2, 3\}$*, with* 2*,* 3 *and* 5 *seats respectively and a majority quota of* 6*. The winning coalitions are* $\{1, 3\}$*,* $\{2, 3\}$ *and*

$\{1, 2, 3\}$.

The decisiveness index of the game is

$$\delta(N, W) = \frac{3}{8}$$

and the loose protectionism index is

$$\delta^*(N, W) = \frac{5}{8}.$$

We evaluate now the decisiveness index and the loose protectionism index of the parties

$$\delta_1(N, W) = \tfrac{1}{2} \quad \delta_2(N, W) = \tfrac{1}{2} \quad \delta_3(N, W) = \tfrac{3}{4}$$

$$\delta_1^*(N, W) = \tfrac{3}{4} \quad \delta_2^*(N, W) = \tfrac{3}{4} \quad \delta_3^*(N, W) = 1$$

We observe that player 1 has full veto power being a veto player, while no player is a dictator. This is an example of a weak and proper game, then the loose protectionism indices of the players are greater than their decisiveness indices.

Evaluating now the Banzhaf strict protectionism index and Johnston index, we obtain

$$\rho_1(N, W) = \tfrac{1}{4} \quad \rho_2(N, W) = \tfrac{1}{4} \quad \rho_3(N, W) = \tfrac{1}{4}$$

$$J_1(N, W) = \tfrac{1}{6} \quad J_2(N, W) = \tfrac{1}{6} \quad J_3(N, W) = \tfrac{2}{3}$$

Banzhaf strict protectionism index assigns the same power of blocking to every party, in particular also to party 3, which is a veto player. Johnston index assigns to party 3 four times the power given to the others, while for our index of veto it has only four thirds of the power of the other parties.

The sum of the veto power of the agents may be lower than 1, but this requires that there is a large number of winning coalitions, and consequently a small number of blocking coalitions. A simple situation is represented by a restricted committee that have to decide which proposal may be admitted to a large assembly examination (e.g. a parliamentary commission that have to decide which laws can be discussed by the Parliament). Usually, a very low majority is required, just to avoid to waste time on proposals that are of no interest for anybody. If we suppose that the committee is formed by 7 representatives and it is necessary that at least two of them vote in favor of discussing it we have that the blocking coalitions are those of 6 or 7 players. So the veto power of each person is 7/64 and the total is 49/64.

5 A Bayesian Model

In this section we present a new model, following the idea of Harsanyi [8] of a non-cooperative game in which the players do not have complete information of the game itself. In this way we are able to extend the model to the situation in which the agents do not vote in favor or against a proposal with probability 1/2, but with a different probability distribution. The lacking of information is due to the fact that the agents can predict the preferences and, consequently, the behavior of the other players, but they cannot be sure about it. Then, they can then only evaluate an expected payoff, trying to maximize it.

A *game with incomplete information played by bayesian players* , or simply a *bayesian game*, is a 5-tuple $(N, \{C_i\}_{i\in N}, \{T_i\}_{i\in N}, \{p_{ik}\}_{i\in N,\ k\in T_i}, \{u_i\}_{i\in N})$ where

- N is the set of players;
- C_i is the set of the actions of player i;
- T_i is the set of types of player i;
- p_{ik} is the probability of player i of being of type k, with $k \in T_i$, $\sum_{k\in T_i} p_{ik} = 1$;
- $u_i : \prod_{j\in N} C_j \times \prod_{j\in N} T_j \to \mathbb{R}$ is the utility function of player i.

A pure strategy for player i is a function $s_i : T_i \to C_i$ and Σ_i is the set of all the pure strategies of i. A mixed strategy for player i is a function $\sigma_i : C_i \times T_i \to [0, 1]$ with $\sum_{c\in C_i} \sigma_i(c, t) = 1$ for each $t \in T_i$.

In Example 1, $N = \{1, 2, 3\}$ is the set of players, i.e. the parties of the Parliament. Adopting a non-cooperative approach, we assume that the parties, instead of cooperating, vote independently. Each one has two choices: voting yes (Y) or voting no (N), then $C_i = \{Y, N\}$ for each $i \in N$. The types of the parties can be identified with the ideological position about the being in favor of the proposal (P) or in favor of the status quo (Q), then $T_i = \{P, Q\}$ for each $i \in N$. A given probability is assigned to the types of the players, in our model equal to 1/2, then $p_{ik} = 1/2$ for each $i \in N$, $k \in T_i$. These probabilities may represent, more in general, the prediction each type of each player does on the possibility of the other players of being of a certain type, but we take a simplified situation in which they are all equal and given a priori; in our example, every voter knows that every other player can be of type in favor or of type against the proposal with probability 1/2. The outcome of the game is given by “the law is approved”, if the parties which voted Y have total number of seats greater than of equal to the majority quota, “the law is not approved” otherwise. The payoff of each party is 1 if it is of type P and the law is approved or if it is of type Q and the law is not approved, 0 otherwise. Formally

$$u_i(s_1,\ldots,s_n) = \begin{cases} 1 & \text{if } T_i = P \text{ and } \sum_{j\in N: s_j(T_j)=Y} w_j \geq q \\ 1 & \text{if } T_i = Q \text{ and } \sum_{j\in N: s_j(T_j)=Y} w_j < q \\ 0 & \text{otherwise} \end{cases}$$

In Table 1 we represent the game in strategic form, where the payoffs of the parties are shown in the 8 different configurations, starting from when they are all of type P, in favor of the proposal, finishing with the case of when they are all of type Q, in favor of the status quo. In every situation, that we call *state of nature*, we assume that party 1 chooses the row, 2 the column and 3 the matrix. The first choice for all of them corresponds to voting Y, the second one to voting N. Each player has a utility of 1 when the outcome is consistent with the type of the player who has been selected, 0 otherwise.

Table 1: strategic form of the game

$$
\begin{array}{r||c}
(1_P, 2_P, 3_P) & \begin{pmatrix} (1,1,1) & (1,1,1) \\ (1,1,1) & (0,0,0) \end{pmatrix} \quad \begin{pmatrix} (0,0,0) & (0,0,0) \\ (0,0,0) & (0,0,0) \end{pmatrix} \\
 & \\
(1_P, 2_P, 3_Q) & \begin{pmatrix} (1,1,0) & (1,1,0) \\ (1,1,0) & (0,0,1) \end{pmatrix} \quad \begin{pmatrix} (0,0,1) & (0,0,1) \\ (0,0,1) & (0,0,1) \end{pmatrix} \\
 & \\
(1_P, 2_Q, 3_P) & \begin{pmatrix} (1,0,1) & (1,0,1) \\ (1,0,1) & (0,1,0) \end{pmatrix} \quad \begin{pmatrix} (0,1,0) & (0,1,0) \\ (0,1,0) & (0,1,0) \end{pmatrix} \\
 & \\
(1_P, 2_Q, 3_P) & \begin{pmatrix} (1,0,0) & (1,0,0) \\ (1,0,0) & (0,1,1) \end{pmatrix} \quad \begin{pmatrix} (0,1,1) & (0,1,1) \\ (0,1,1) & (0,1,1) \end{pmatrix} \\
 & \\
(1_Q, 2_P, 3_P) & \begin{pmatrix} (0,1,1) & (0,1,1) \\ (0,1,1) & (1,0,0) \end{pmatrix} \quad \begin{pmatrix} (1,0,0) & (1,0,0) \\ (1,0,0) & (1,0,0) \end{pmatrix} \\
 & \\
(1_Q, 2_P, 3_Q) & \begin{pmatrix} (0,1,0) & (0,1,0) \\ (0,1,0) & (1,0,1) \end{pmatrix} \quad \begin{pmatrix} (1,0,1) & (1,0,1) \\ (1,0,1) & (1,0,1) \end{pmatrix} \\
 & \\
(1_Q, 2_Q, 3_P) & \begin{pmatrix} (0,0,1) & (0,0,1) \\ (0,0,1) & (1,1,0) \end{pmatrix} \quad \begin{pmatrix} (1,1,0) & (1,1,0) \\ (1,1,0) & (1,1,0) \end{pmatrix} \\
 & \\
(1_Q, 2_Q, 3_Q) & \begin{pmatrix} (0,0,0) & (0,0,0) \\ (0,0,0) & (1,1,1) \end{pmatrix} \quad \begin{pmatrix} (1,1,1) & (1,1,1) \\ (1,1,1) & (1,1,1) \end{pmatrix}
\end{array}
$$

In order to show the complexity of the problem, we show the situation in extensive form in Figure 1, where the black dots represent the choices of the nature which selects the type of each party with probability 1/2. Then each player (the white dots) has to take its own decision, selecting an action between Y and N, finally one of the 64 outcomes is selected.

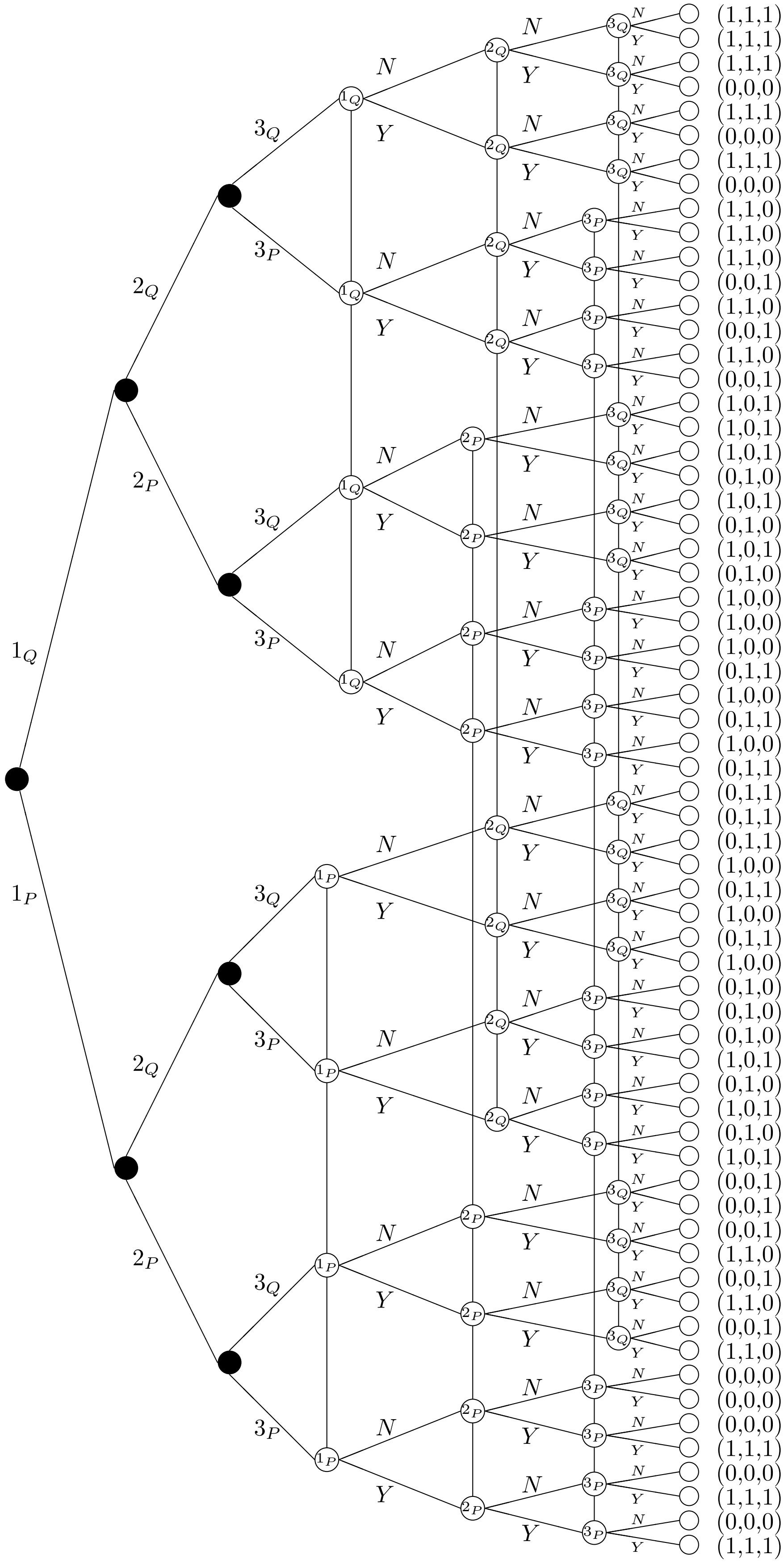

Figure 1: extensive form of the game

Every party knows its own type, but not the types of the other two parties. As it gives probability 1/2 to every type of the other parties, it assigns probability 1/4 to be in a given state of nature. If party 1, for example, is of type P, it will give probability 1/4 to each one of the first four states of nature shown in Table 1. Assuming that party 2 will play $(p, 1-p)$ and party 3 $(q, 1-q)$, the expected payoff for player 1 when s/he plays Y is

$$\frac{1}{4}[pq+(1-p)q]+\frac{1}{4}[pq+(1-p)q]+\frac{1}{4}[pq+(1-p)q]+\frac{1}{4}[pq+(1-p)q]=q$$

and when s/he plays N is

$$\frac{1}{4}[pq]+\frac{1}{4}[pq]+\frac{1}{4}[pq]+\frac{1}{4}[pq]=pq$$

then the best choice for party 1 of type P is to play $(t, 1-t)$ with $t = 1$ if $p < 1$ and $t \in [0, 1]$ if $p = 1$.

Writing the best reply for every type of every player, we obtain the obvious result that the optimal strategy for the players is to choose Y if they are of type P and N if they are of type Q. This is the *Bayesian pure equilibrium* of the game.

The interesting result is that, when the probabilities of the types are all equal to 1/2, playing the equilibrium strategy every party of type P can obtain an expected utility equal to its decisiveness index and every party of type Q an expected utility equal to its loose protectionism index.

6 Concluding Remarks

In this paper we proposed a new quantitative index for measuring the veto power of each agent in a decisional situation. Our main aim was to evaluate the veto power starting from the remark that a classical veto player, i.e. an agent whose approval is required for passing a proposal, has full veto power, that we assume equal to one; consequently, the power of the other agents represents the fraction of veto power they have in comparison with the full veto power and the sum of the power of all the agents has not to be equal to one.

Referring to the decisiveness and loose protectionism indices introduced by Carreras [2] for evaluating the characteristics of the game as a whole, we introduced the decisiveness and loose protectionism indices for a player, extending the results of Carreras, in order to have a measure of the role of each player, analyzing some properties of these two indices.

Then, we proposed a more specific index that takes into account from a quantitative point of view the influence of a player in rejecting a proposal when s/he votes against it. This index simply counts the frequency of the situations in which the final outcome is negative when her/his choice is negative. This approach provides a general evaluation of the veto power, in any possible decision ballot. It is clear that in a more particular setting, i.e. the vote on a given proposal the behavior and the preferences of each decision-maker may be forecasted on the basis of previous analogous situations or on specific information that the agents got in the past. We decided to represent this situation as a Bayesian game in which the agents may have two types, in favor of the proposal or in favor of the *status quo*. In Section 5 we assigned to each agent a fixed probability for each type, in order to have a unique representation of the game; of course when they play the game, the agents may introduce in the model the probabilities that better represent their beliefs on the types of the other players, accounting their private information, their personal experience and expertise, and their knowledge of the situation at hand. We want to stress that the computationally simple loose protectionism index of the players coincides with the Bayesian equilibrium of the Bayesian game when the probabilities of all the types of each player are equal to 1/2.

This work is a step forward in the evaluation of the veto power of the decision-makers, but other features of the problem may be introduced in the model in order to have a more suitable measure of their power.

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