A quantitative evaluation of veto power

Vito (Franco) Fragnelli
Università del Piemonte Orientale
vito.fragnelli@uniupo.it

M.Chessa, V.Fragnelli - A Quantitative Evaluation of Veto Power - Operations Research and Decisions 21 : 5-19 - 2011

Game Theory and Voting Systems

Lake Como School of Advanced Studies, 3-8 September 2017 (Campione d'Italia)

Contents

1 Introduction and Aims

The United Nations Security Council (UNSC) represents the most typical example of veto power
The Council is composed of 5 permanent members (China, France, Russian Federation, United Kingdom and United States) and 10 non-permanent members; each member has one vote
Great Power Unanimity: Decisions on substantive matters require 9 votes, and the votes of all 5 permanent members
The permanent members are called *veto players*

Mercik (2011a) observes that the right of veto will increase the power of a player in most cases

According to Tsebelis (2002), *a veto player is an individual or collective actor whose agreement is necessary for policy changes*

A natural question is: how to evaluate the veto power?

More precisely

- Are veto power and power analogous concepts?
- May we evaluate them with the same instruments?

The existing power indices take into account different features

- the order of the players entering a winning coalitions (Shapley-Shubik index, 1954)
- the winning coalitions (Banzhaf-Coleman index, Banzhaf, 1965 and Coleman, 1971)
- the minimal winning coalitions (Deegan-Packel index, 1978 and Public Good Index, Holler, 1982)
- the quasi-minimal winning coalitions (Johnston index, 1978)
- the possible weights of the players (Weighted Shapley value, Kalai and Samet, 1987)
- the possible coalition structures (Owen, 1977 and Winter, 1989)
- the possible connections between the players (Myerson index, 1977 and FP indices Fragnelli, Ottone and Sattanino, 2009 and Chessa and Fragnelli, 2011)

A suitable index for analyzing the power of veto should be different

A party can be able alone to block a proposal, but it may not have the possibility to make an opposite law

In order to block a proposal it is no longer necessary to have a common ideological position, vanishing the concept of coalition structure and connection

Carreras (2005 and 2009) analyzes the blocking power in a simple game, both from a collective and from an individual point of view, introducing the *strict protectionism index* and the *Banzhaf strict blocking power index*
Ishikawa and Inohara (2007) formalize the *blockability relation*
Kitamura and Inohara (2009) propose the *blockability index*
Mercik (2011b) suggests the evaluation of the power of veto starting from Johnston power index, after defining some suitable axioms for an index for veto

HERE

A quantitative evaluation of the power of veto

It is no longer necessary that the power of the agents sum up to 1, as all the veto players have full power to block a proposal

The new index assigns full veto power (for simplicity equal to 1) to all those agents who are able alone to veto a proposal; all the other players get a non-negative veto power smaller than 1 according to their possibility to stop an approval joining other players

2 Preliminaries

A *cooperative TU game* is a pair (N, v) where $N = \{1, 2, \ldots, n\}$ is a non-empty finite set of players and $v : 2^N \to \mathbb{R}, v(\varnothing) = 0$ is the *characteristic function* which associates with every non-empty subset S of N, called *coalition*, a real number $v(S)$, which represents the *worth* of S
(N, v) is *simple* if $v(S) \in \{0, 1\}$ for each $S \subseteq N$, $v(N) = 1$ and given $S, T \subseteq N$, $S \subseteq T \Rightarrow v(S) \leq v(T)$. When $v(S) = 1$, the coalition is *winning*, *losing* otherwise
Let W be the set of all the winning coalitions
A player i is a *veto player* if s/he belongs to all winning coalitions
A player i is a *dictator* if $\{i\} \in W$
Given a coalition $S \in W$, a player $i \in S$ is *critical* for S if $S \setminus \{i\} \notin W$
The quantity $v(S) - v(S \setminus \{i\})$ is called the *marginal contribution of player* i *w.r.t.* S
A winning coalition is *minimal* if each proper subcoalition is losing; W^m denotes the set of all the minimal winning coalitions

Political situations are frequently described through weighted majority situations

A *weighted majority situation* is a triple (N, w, q)

where N is the set of agents, e.g. the parties of a Parliament

$w = (w_1, w_2, \ldots, w_n)$ is the *vector of weights*

q is the *majority quota*

A simpler notation is $[q; w_1, w_2, \ldots, w_n]$

The corresponding *weighted majority game* is (N, w), with $w : 2^N \rightarrow \{0, 1\}$

$$w(S) = \begin{cases} 1 \text{ if } \sum_{j \in S} w_j \geq q \\ 0 \text{ otherwise} \end{cases} \quad \forall S \subseteq N$$

A simple game can always be described by the set W (or simply W^m), while it is not always possible to represent it as a weighted majority situation

3 Decisiveness and loose protectionism indices

Carreras (2005) provides a numerical measure of the agility of the collective decision-making mechanism

Given a simple game (N, v), the set of coalitions 2^N splits into four classes:

- D (*decisive winning*): $S \in W$ such that $N \setminus S \notin W$
- C (*conflictive winning*): $S \in W$ such that $N \setminus S \in W$
- Q (*blocking*): $S \notin W$ such that $N \setminus S \notin W$
- P (*strictly losing*): $S \notin W$ such that $N \setminus S \in W$

Thus, $W = D \cup C$

The game is *strong* if $Q = \varnothing$ and *weak* otherwise

The game is *proper* if $C = \varnothing$ and *improper* otherwise

A proper and strong game is called *decisive*

Carreras defines the *decisiveness index*

$$\delta(N,W) = \frac{|W|}{2^n}$$

i.e. the probability that an abstract proposal will pass in (N,W), when each agent may vote in favor (Y) or against (N) with probability $1/2$
The probabilities of the agents are assumed to be independent
$0 < \delta(N,W) < 1$ as $\varnothing \notin W$ and $N \in W$

Given two simple games with the same player set, $|W| < |W'|$ implies $\delta(N,W) < \delta(N,W')$
If a game is decisive, then $\delta(N,W) = 1/2$; if it is weak and proper, then $\delta(N,W) < 1/2$; if it is strong and improper, then $\delta(N,W) > 1/2$

Let (N, W) be a simple game and (N, W^*) be the dual game where $W^* = \{S \subseteq N : N \setminus S \notin W\}$

- $\delta(N, W^*) + \delta(N, W) = 1$
- $\delta(N, W^*) - \delta(N, W) = \dfrac{|Q| - |C|}{2^n}$

Carreras (2005) defines the *loose protectionism index*, that provides a numerical measure of the inertia of any decision-making mechanism

$$\delta^*(N, W) = \delta(N, W^*) = 1 - \delta(N, W)$$

i.e. the probability that a proposal will not pass in (N, W), when again each agent may vote in favor or against with probability $1/2$

As $\delta^*(N, W) = 1 - \delta(N, W)$, Carreras (2009) observes that it does not provide any new information, and introduces the *strict protectionism index*

$$\pi(N, W) = \frac{|Q|}{2^n}$$

Then he defines a *blocking swing* for player $i \in N$ as a pair $(S, S \setminus \{i\})$ s.t. $S \in Q$ and $S \setminus \{i\} \notin Q$ and the *Banzhaf strict protectionism index*, an individual blocking power index

$$\rho_i(N, W) = \xi_i(N, W)/2^{n-1}$$

where $\xi_i(N, W)$ is the number of blocking swings for i

4 Decisiveness and loose protectionism indices of a player

From the perspective of each player, the decisiveness of a game may be viewed as the probability that a proposal will pass in (N, W) when player $i \in N$ votes in favor of the proposal (Y) and the other players vote in favor or against with probability $1/2$

Let W_i be the set of winning coalitions including player i, i.e. $W_i = \{S \in W : i \in S\}$

Definition 1. *The* decisiveness index of player i *is defined as*

$$\delta_i(N, W) = \frac{|W_i|}{2^{n-1}} \tag{1}$$

$0 < \delta_i(N, W) \leq 1$ as $N \in W_i$

If A stands for "the proposal will pass" and B for "player i votes in favor", the index corresponds to the conditional probability $P(A|B) = \frac{P(A \cap B)}{P(B)}$

Proposition 1. *When i is a veto player,* $\delta_i(N, W) = 2\delta(N, W)$

Similarly, it is possible to consider the probability that a proposal will not pass in (N, W) when player i votes against the proposal (N) and the others vote in favor or against with probability $1/2$

Definition 2. *The* loose protectionism index of player i *is defined as*

$$\delta_i^*(N, W) = \frac{2^{n-1} - |W| + |W_i|}{2^{n-1}} = 1 - 2\delta(N, W) + \delta_i(N, W) \tag{2}$$

$0 < \delta_i^*(N, W) \leq 1$ as $\varnothing \notin W$

The numerator counts the number of losing coalitions which do not include player i

Remark 1. *Player i is a dictator iff $\delta_i(N,W)=1$ and is of veto iff $\delta_i^*(N,W)=1$*

$\delta_i(N,W)$ and $\delta_i^*(N,W)$ are strictly related and this relation depends on the decisiveness index of the game. In particular

- *if the game is weak and proper, as $\delta(N,W)<1/2$, we get that $\delta_i^*(N,W)>\delta_i(N,W)$ for each $i\in N$*
- *if the game is strong and improper, by duality $\delta(N,W)>1/2$ and $\delta_i^*(N,W)<\delta_i(N,W)$ for each $i\in N$*
- *if the game is decisive, then a player is of veto ($\delta_i^*(N,W)=1$) iff it is a dictator ($\delta_i(N,W)=1$); in general, $\delta_i(N,W)=\delta_i^*(N,W)$ for each $i\in N$*

The two indices are also directly related as stated in the following proposition

Proposition 2. *$\delta_i^*(N,W)=\delta_i(N,W^*)$, for every $i\in N$*

Carreras (2005) observes that the Banzhaf index also measures the decisiveness of a game from the perspective of each player. In particular, let (N, W) be a game and $i \in N$, then

$$\beta_i(N, W) = 2\delta(N, W) - 2\delta(N_{-\{i\}}, W_{-\{i\}})$$

where $(N_{-\{i\}}, W_{-\{i\}})$ denotes the residual game after that player i leaves

This basic relationship between the decisiveness index and the Banzhaf index suggests to look for a possible relation between the Banzhaf index and the indices defined in (1) and (2)
When $i \in N$ is a veto player, as Carreras noticed $\beta_i(N, W) = 2\delta(N, W)$, then by Proposition 1, $\beta_i(N, W) = \delta_i(N, W)$

The indices in (1) and (2) are quantitative indices and they evaluate the power of a player in making a proposal been accepted (δ_i) or rejected (δ_i^*)

Example 1 gives the previous indices for a simple theoretical situation. For sake of completeness, also the Banzhaf strict protectionism index, ρ, and the Johnston index, J, are reported
The comparison is carried out using the ratios of the indices among the players, as not all the indices sum up to one

Example 1. *Consider the simple weighted majority situation* $[6;2,3,5]$*. The winning coalitions are* $\{1,3\}$, $\{2,3\}$ *and* $\{1,2,3\}$
The decisiveness index and the loose protectionism index of the game are

$$\delta(N,W)=\tfrac{3}{8};\ \delta^*(N,W)=\tfrac{5}{8}$$

The decisiveness index and the loose protectionism index of the parties are

$$\delta_1(N,W)=\tfrac{1}{2};\ \delta_2(N,W)=\tfrac{1}{2};\ \delta_3(N,W)=\tfrac{3}{4}$$
$$\delta_1^*(N,W)=\tfrac{3}{4};\ \delta_2^*(N,W)=\tfrac{3}{4};\ \delta_3^*(N,W)=1$$

Party 3 has full veto power being a veto player, while no player is a dictator. This is an example of a weak and proper game, then the loose protectionism indices of the players are greater than their decisiveness indices
The Banzhaf strict protectionism index and Johnston index are

$$\rho_1(N,W)=\tfrac{1}{4};\ \rho_2(N,W)=\tfrac{1}{4};\ \rho_3(N,W)=\tfrac{1}{4}$$
$$J_1(N,W)=\tfrac{1}{6};\ J_2(N,W)=\tfrac{1}{6};\ J_3(N,W)=\tfrac{2}{3}$$

Banzhaf strict protectionism index assigns the same power of blocking to every party
Johnston index assigns to party 3 four times the power given to the others, while for our index of veto it has only four thirds of the power of the other parties

The sum of the veto power of the agents may be lower than 1, but this requires that there is a large number of winning coalitions, and consequently a small number of blocking coalitions
A simple situation is represented by a restricted committee that have to decide which proposal may be admitted to a large assembly examination (e.g. a parliamentary commission that have to decide which laws can be discussed by the Parliament). Usually, a very low majority is required, just to avoid to waste time on proposals that are of no interest for anybody. Supposing that the committee is formed by 7 representatives and at least two of them have to vote in favor of discussing it, the blocking coalitions are those of 6 or 7 players. So the veto power of each person is $7/64$ and the total is $49/64$

5 A Bayesian Model

The following model makes use of a non-cooperative game with incomplete information (Harsanyi, 1967)

This approach allows extending the model to the situation in which the agents vote in favor or against a proposal with any probability distribution

The agents can predict the preferences and, consequently, the behavior of the other players, but they cannot be sure about it, so they can only maximize the expected payoff

A *Bayesian game*, is a 5-tuple $(N, \{C_i\}_{i\in N}, \{T_i\}_{i\in N}, \{p_{ik}\}_{i\in N,\ k\in T_i}, \{u_i\}_{i\in N})$

where N is the set of players

C_i is the set of the actions of player i

T_i is the set of types of player i

p_{ik} is the probability of player i of being of type k, with $k \in T_i$, $\sum_{k\in T_i} p_{ik} = 1$

$u_i : \prod_{j\in N} C_j \times \prod_{j\in N} T_j \to \mathbb{R}$ is the utility function of player i

A pure strategy for player i is a function $s_i : T_i \to C_i$ and Σ_i is the set of all the pure strategies of i. A mixed strategy for player i is a function $\sigma_i : C_i \times T_i \to [0,1]$ with $\sum_{c\in C_i} \sigma_i(c,t) = 1$ for each $t \in T_i$

Referring to Example 1, adopting a Bayesian approach, the parties vote independently, instead of cooperating

Each one has two choices: voting yes (Y) or voting no (N), then $C_i = \{\text{Y, N}\},\ i \in N$

The types of the parties can be identified with the ideological position in favor of the proposal (P) or in favor of the status quo (Q), then $T_i = \{P, Q\},\ i \in N$

The probability assigned to the types of the players is $1/2$, then $p_{ik} = 1/2$ for each $i \in N, k \in T_i$, i.e. every voter knows that every other player can be of type P or Q with probability $1/2$ (more complex choices are possible)

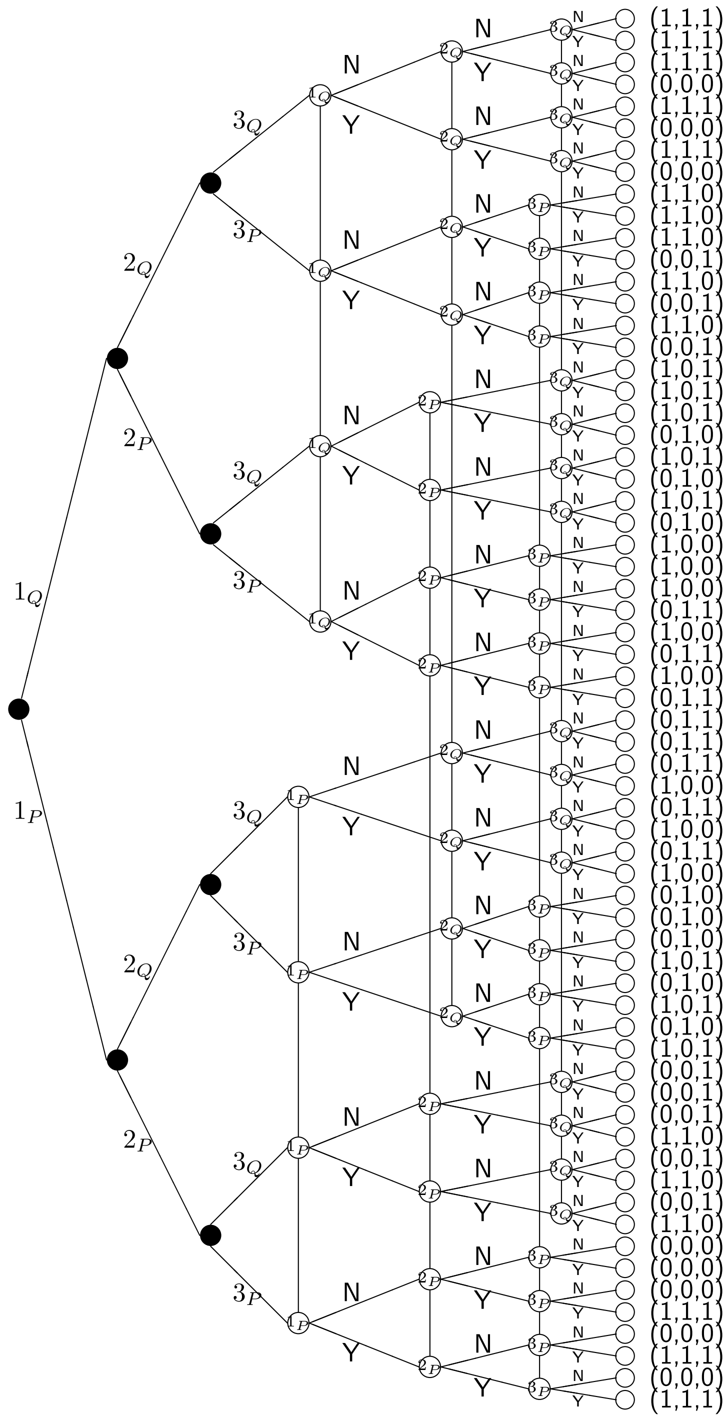
The outcome of the game follows the simple majority rule

The payoff of each party is 1 if it is of type P and the law is approved or if it is of type Q and the law is not approved, 0 otherwise

The following table represents the game in strategic form. In every *state of nature*, party 1 chooses the row, 2 the column and 3 the matrix. For all of them, the first choice corresponds Y, the second one to N

In the extensive form, the black dots represent the choices of the nature which selects the type of each party with probability $1/2$. Then each player (the white dots) has to take its own decision between Y and N

$(1_P, 2_P, 3_P)$	$\begin{pmatrix} (1,1,1) & (1,1,1) \\ (1,1,1) & (0,0,0) \end{pmatrix}$	$\begin{pmatrix} (0,0,0) & (0,0,0) \\ (0,0,0) & (0,0,0) \end{pmatrix}$
$(1_P, 2_P, 3_Q)$	$\begin{pmatrix} (1,1,0) & (1,1,0) \\ (1,1,0) & (0,0,1) \end{pmatrix}$	$\begin{pmatrix} (0,0,1) & (0,0,1) \\ (0,0,1) & (0,0,1) \end{pmatrix}$
$(1_P, 2_Q, 3_P)$	$\begin{pmatrix} (1,0,1) & (1,0,1) \\ (1,0,1) & (0,1,0) \end{pmatrix}$	$\begin{pmatrix} (0,1,0) & (0,1,0) \\ (0,1,0) & (0,1,0) \end{pmatrix}$
$(1_P, 2_Q, 3_P)$	$\begin{pmatrix} (1,0,0) & (1,0,0) \\ (1,0,0) & (0,1,1) \end{pmatrix}$	$\begin{pmatrix} (0,1,1) & (0,1,1) \\ (0,1,1) & (0,1,1) \end{pmatrix}$
$(1_Q, 2_P, 3_P)$	$\begin{pmatrix} (0,1,1) & (0,1,1) \\ (0,1,1) & (1,0,0) \end{pmatrix}$	$\begin{pmatrix} (1,0,0) & (1,0,0) \\ (1,0,0) & (1,0,0) \end{pmatrix}$
$(1_Q, 2_P, 3_Q)$	$\begin{pmatrix} (0,1,0) & (0,1,0) \\ (0,1,0) & (1,0,1) \end{pmatrix}$	$\begin{pmatrix} (1,0,1) & (1,0,1) \\ (1,0,1) & (1,0,1) \end{pmatrix}$
$(1_Q, 2_Q, 3_P)$	$\begin{pmatrix} (0,0,1) & (0,0,1) \\ (0,0,1) & (1,1,0) \end{pmatrix}$	$\begin{pmatrix} (1,1,0) & (1,1,0) \\ (1,1,0) & (1,1,0) \end{pmatrix}$
$(1_Q, 2_Q, 3_Q)$	$\begin{pmatrix} (0,0,0) & (0,0,0) \\ (0,0,0) & (1,1,1) \end{pmatrix}$	$\begin{pmatrix} (1,1,1) & (1,1,1) \\ (1,1,1) & (1,1,1) \end{pmatrix}$



Every party knows its own type, but not the types of the other two parties. As it gives probability $1/2$ to every type of the other parties, it assigns probability $1/4$ to be in a given state of nature Suppose that party 1 is of type P; assuming that party 2 plays $(p, 1-p)$ and party 3 $(q, 1-q)$, the expected payoff for party 1 playing Y is

$$\frac{1}{4}[pq + (1-p)q] + \frac{1}{4}[pq + (1-p)q] + \frac{1}{4}[pq + (1-p)q] + \frac{1}{4}[pq + (1-p)q] = q$$

and playing N is

$$\frac{1}{4}[pq] + \frac{1}{4}[pq] + \frac{1}{4}[pq] + \frac{1}{4}[pq] = pq$$

then the best choice for party 1 of type P is to play $(t, 1-t)$ with $t = 1$ if $p < 1$ and $t \in [0, 1]$ if $p = 1$

Obviously, the optimal strategy for the parties is to choose Y if they are of type P, and N if they are of type Q. This is the *Bayesian pure equilibrium* of the game

The interesting result is that, when the probabilities of the types are all equal to $1/2$, playing the equilibrium strategy every party of type P can obtain an expected utility equal to its decisiveness index and every party of type Q an expected utility equal to its loose protectionism index

References

[1] BANZHAF J.F., Weighted voting doesn't work: a mathematical analysis, Rutgers Law Review, 1965, Vol. 19, pp. 317-343.

[2] CARRERAS F., A decisiveness index for simple games, European Journal of Operational Research, 2005, Vol. 163, pp. 370-387.

[3] CARRERAS F. Protectionism and blocking power indices, Top, 2009, Vol. 17, pp. 70-84.

[4] CHESSA M., FRAGNELLI V., Embedding classical indices in the FP family, AUCO Czech Economic Review, 2011, Vol. 5, pp. 289-305.

[5] COLEMAN J.S., Control of Collectivities and the Power of a Collectivity to Act, [in:] Lieberman, B. (ed), Social Choice, Gordon and Breach, London, 1971. pp. 269-300

[6] DEEGAN J., PACKEL E.W., A New Index of Power for Simple n-person Games, International Journal of Game Theory, 1978, Vol. 7, pp. 113-123.

[7] FRAGNELLI V., OTTONE S., SATTANINO R., A new family of power indices for voting games, Homo Oeconomicus, 2009, Vol. 26, pp. 381-394.

[8] HARSANYI J.C., Games with incomplete information played by bayesian players, I-III, Management Science, 1967, Vol. 14, pp. 159-182.

[9] HOLLER M.J., Forming Coalitions and Measuring Voting Power, Political Studies, 1982, Vol. 30, pp. 262-271.

[10] ISHIKAWA K., INOHARA T., A method to compare influence of coalitions on group decision other than desirability relation, Applied Mathematics and Computation, 2007, Vol. 188, pp. 838-849.

[11] JOHNSTON R.J., On the measurement of power: some reactions to Laver, Environment and Planning A, 1978, Vol. 10, pp. 907-914.

[12] KALAI E., SAMET D., On weighted Shapley values, International Journal of Game Theory, 1987, Vol. 16, pp. 205-222.

[13] KITAMURA M., INOHARA T., An Extended Power Index to Evaluate Coalition Influence Based on Blockability Relations on Simple Games, in Proceedings of the 2009 IEEE International Conference on Systems, Man, and Cybernetics, San Antonio, TX, USA, 2009, pp. 1527-1532.

[14] MERCIK J., On a priory Evaluation of Power of Veto, [in:] Herrera-Viedma, E., García-Lapresta, J.L., Kacprzyk, J., Fedrizzi, M., Nurmi, H. and Zadrożny, S. (eds), Consensual Processes, Springer Verlag, Berlin-Heidelberg, 2011a. pp. 145–156

[15] MERCIK J., On Axiomatization and Measuring of Power of Veto, in Proceedings 7th Spain-Italy-Netherlands Meeting on Game Theory, Paris, 2011b.

[16] MYERSON R., Graphs and Cooperation in Games, Mathematics of Operations Research, 1977, Vol. 2, pp. 225-229.

[17] OWEN G., Values of Games with a Priori Unions, Lecture Notes in Economic and Mathematical Systems, 1977, Vol. 141, pp. 76–88.

[18] SHAPLEY L.S., SHUBIK M., A method for evaluating the distribution of power in a committee system, The American Political Science Review, 1954, Vol. 48, pp. 787-792.

[19] TSEBELIS G., Veto Players. How Political Institutions Work, NJ: Princeton University Press, 2002, Princeton.

[20] WINTER E., A Value for Cooperative Games with Levels Structure of Cooperation, International Journal of Game Theory, 1989, Vol. 18, pp. 227-240.

Thanks!